RATIONAL CARD #1

→ ***Without using a calculator*** identify all relevant rational features.
→ Then make a sketch (without the calculator) that fits the rational function around these features.

$$f(x) = \frac{2x}{2+x} + 3$$

ADDITIONAL QUESTION: When x → ∞ y → ____

ADDITIONAL QUESTION: When x → –2 y → ____

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ADDITIONAL QUESTION: When x → –2 y → ____

V.A. $x = -2$

H.A. $y = \frac{2}{1} + 3 = 5$

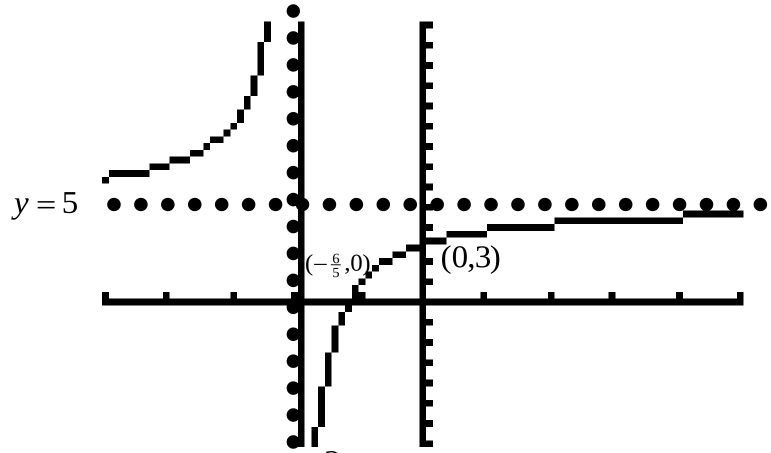
y int $y = \frac{2(0)}{2+0} + 3 = 3 \quad (0,3)$

x int $0 = \frac{2x}{2+x} + 3$

$-3(2+x) = 2x$

$-6 = 5x$

$x = (\frac{-6}{5}, 0)$



Additional Q: y → 5

Additional Q: y → ± ∞

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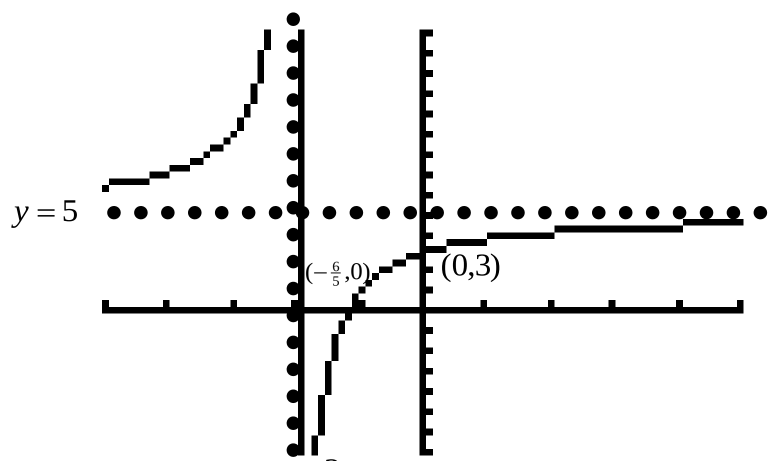

Additional Q: y → 5

Additional Q: y → ± ∞

RATIONAL CARD #2

→ ***Without using a calculator*** identify all relevant rational features.
→ Then make a sketch (without the calculator) that fits the rational function around these features.

$$f(x)=\frac{x+6}{x^2+5x-6}$$

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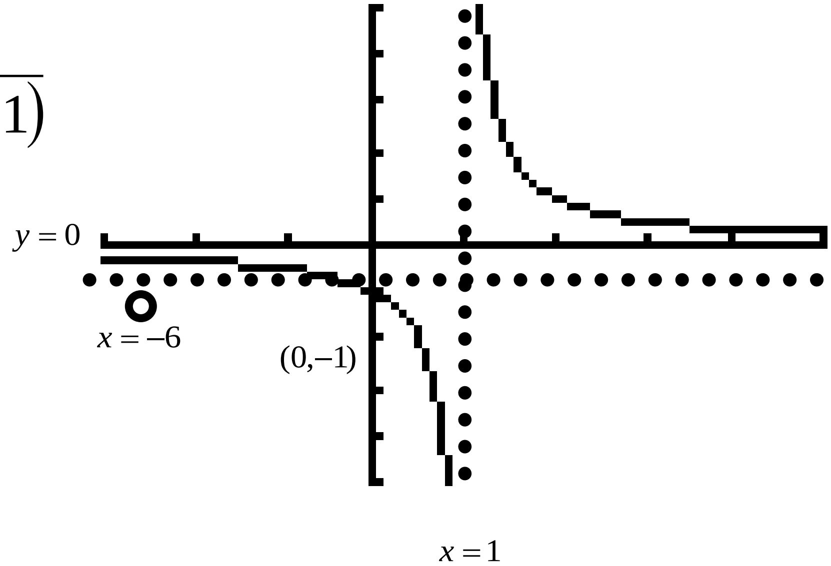
$$f(x)=\frac{x+6}{x^2+5x-6}$$

$$f(x)=\frac{x+6}{x^2+5x-6}=\frac{x+6}{(x+6)(x-1)}$$



Hole $x=-6 \quad y=-\frac{1}{6}$

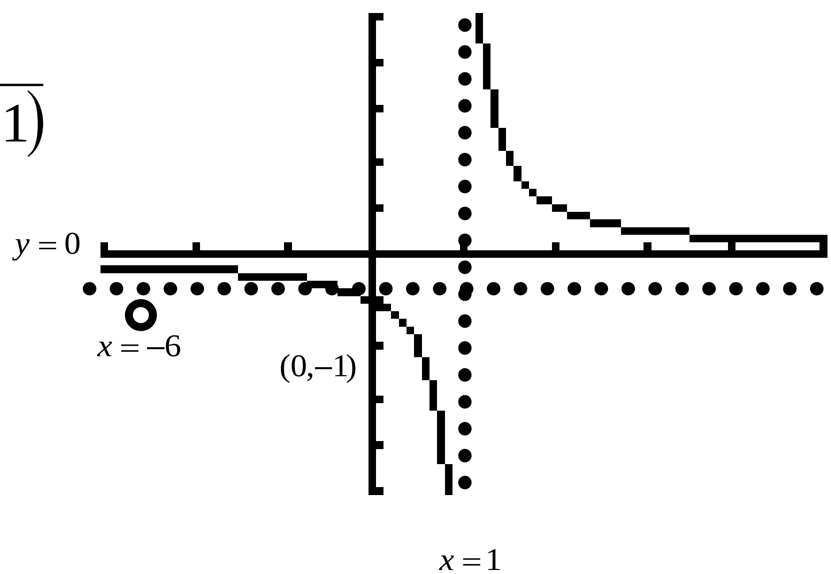
V.A. $x=1$

H.A. $y=0$

y int $y=\frac{0+6}{0^2+5(0)-6}=\frac{6}{-6}=-1 \quad (0,-1)$

x int $0=\frac{x+6}{(x+6)(x-1)}$ can't happen unless x = –6, so NO X INT.

$$f(x)=\frac{x+6}{x^2+5x-6}=\frac{x+6}{(x+6)(x-1)}$$



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RATIONAL CARD #3

→ Identify all relevant rational features.
→ Then make a sketch (***and you MAY use a calculator to help***) that fits the rational function around these features.

$$f(x) = \frac{(x+2)(x-1)}{x(x-2)^2(x+2)}$$

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$$f(x) = \frac{(x+2)(x-1)}{x(x-2)^2(x+2)}$$

Hole $x = -2$ $y = \frac{3}{32}$

V.A. $x = 2$ and $x = 0$

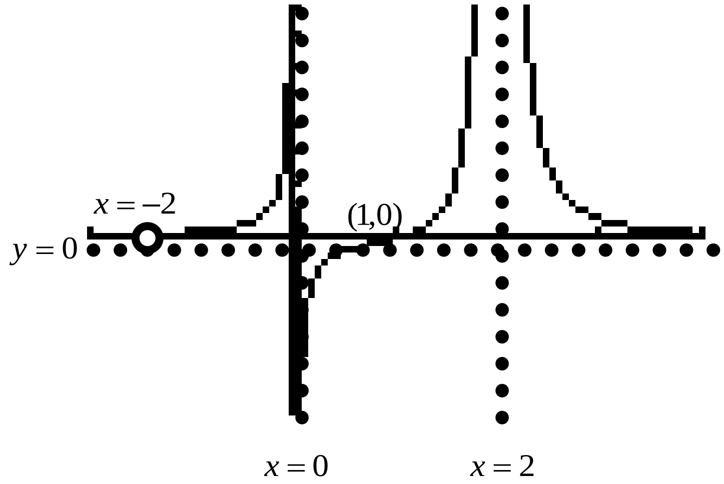
H.A. $y = 0$

y int $y = \frac{(0+2)(0-1)}{0(0-2)(0+2)} = \frac{-2}{0}$ NONE!

x int $0 = \frac{(x+2)(x-1)}{x(x-2)(x+2)}$

$0 = (x+2)(x-1)$

so x must equal –2 or 1
but –2 is already out…
…so x = 1 is only choice (1, 0)



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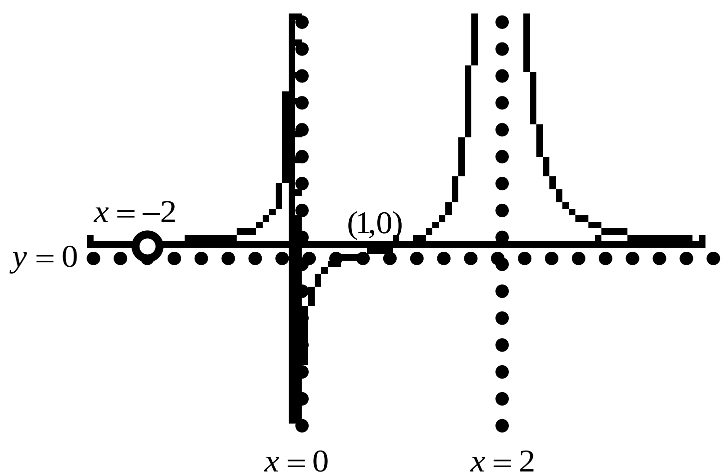

RATIONAL CARD #4

→ Identify all relevant rational features. (x intercept(s) not required)
→ Then make a sketch (***and you MAY use a calculator to help***) that fits the rational function around these features.

$$f(x)=\frac{x^3+2x^2-4x+2}{x^2-4}$$

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$$f(x)=\frac{x^3+2x^2-4x+2}{x^2-4}$$

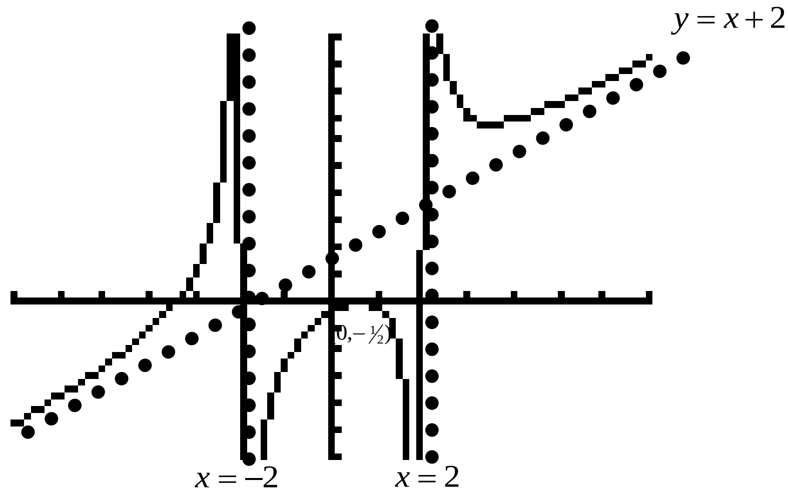
V.A. $x = \pm 2$

H.A. none

Slant $x^2 - 4\overline{\big)x^3 + 2x^2 - 4x + 2}$ gives $y = x + 2$

y int $y = \frac{0^3 + 2(0)^2 - 4(0) + 2}{0^2 - 4} = -\frac{1}{2}$ (0, - ½)

x int none that you need to find without a calculator



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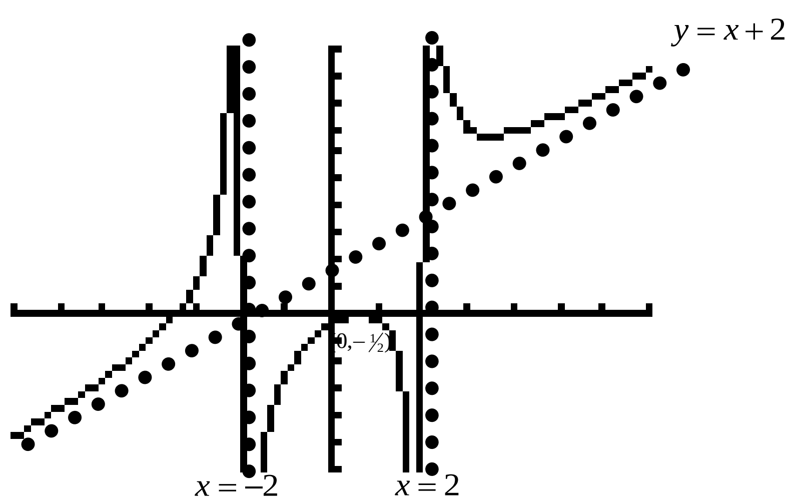

PARABOLA CARD #4

- → Use the "5 point method" to find the vertex, intercepts & a symmetry point.
- → Create an accurate nice sketch of this parabola.
- → This should all be done **WITHOUT a calculator**

$$f(x) = 3x^2 - 12x - 2$$

Arithmetic Help… you should find yourself getting $\sqrt{168}$ during the "5 point method" which equals $2\sqrt{42}$

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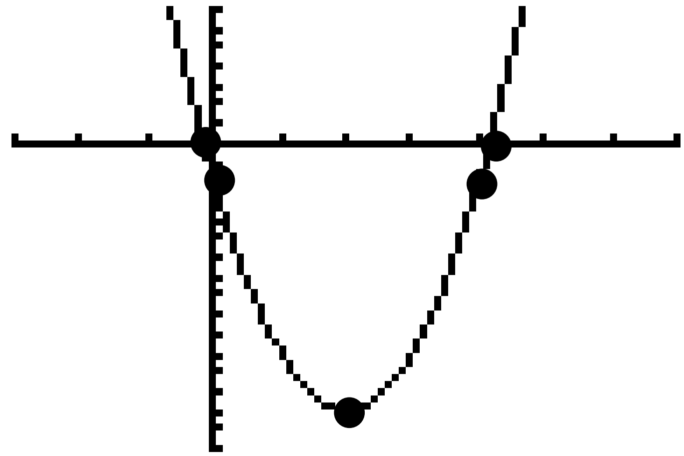
Vertex $x = \frac{-(-12)}{2(3)} = 2$

so $y = 3(2)^2 - 12(2) - 2 = -14$ (2, –14)

y int. (0,–2)

x int $x = \frac{12 \pm \sqrt{144 - 4(3)(-2)}}{2(3)} = \frac{12 \pm 2\sqrt{42}}{6} = 2 \pm \frac{1}{3}\sqrt{42}$

point symmetric with the y intercept: (4,–2)



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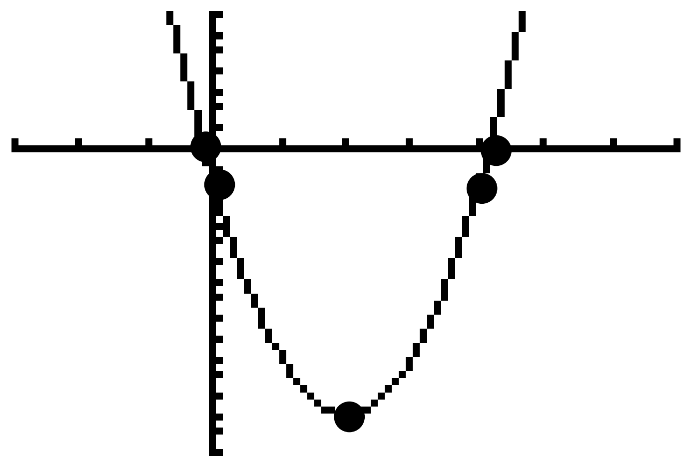

Circle Card #5

→ Remember that it is helpful to write the circle equation in standard form:

$$(x \pm a)^2 + (y \pm b)^2 = r^2$$

→ Identify and label the center.
→ Identify the radius and use it to label one point on the circle
→ Will the final graph be a FULL circle or PART of a circle? (it depends on what you start with)

$$y = \sqrt{9 - x^2} + 6$$

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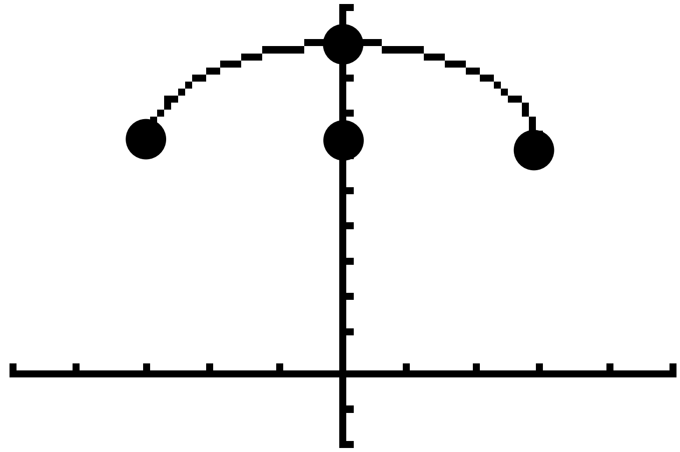
$$y = \sqrt{9 - x^2} + 6$$

Putting the equation in standard form…



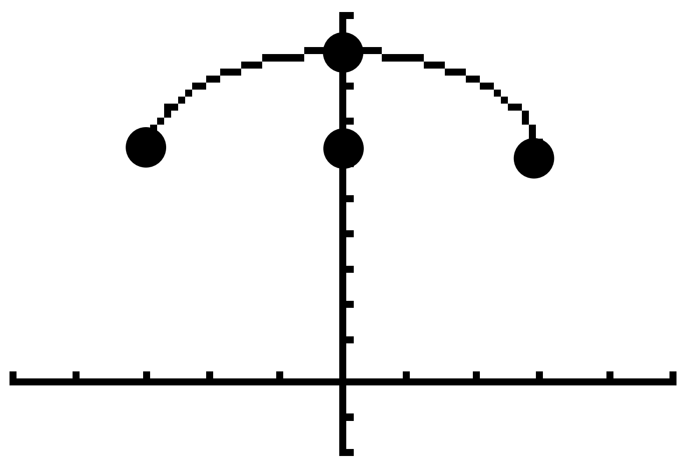
$$y - 6 = \sqrt{9 - x^2}$$
$$(y - 6)^2 = 9 - x^2$$
$$x^2 + (y - 6)^2 = 9 = 3^2$$

We only get the positive ***top*** half of the circle since ***original form*** was given as y = $+\sqrt{}$

So **center** @ **(0, 6)**

Radius = 3
so easy points to find on circle…
(-3, 6) (0, 9) (3, 6)

Putting the equation in standard form…



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Circle Card #6

→ Remember that it is helpful to write the circle equation in standard form:

$$(x \pm a)^2 + (y \pm b)^2 = r^2$$

→ Identify and label the center.
→ Identify the radius and use it to label one point on the circle
→ Will the final graph be a FULL circle or PART of a circle? (it depends on what you start with)

$$\underbrace{x^2 + 6x + 9} + y^2 = 16$$

Hint: There is something special about <u>this</u> group

Circle Card #6

→ Remember that it is helpful to write the circle equation in standard form:

$$(x \pm a)^2 + (y \pm b)^2 = r^2$$

→ Identify and label the center.
→ Identify the radius and use it to label one point on the circle
→ Will the final graph be a FULL circle or PART of a circle? (it depends on what you start with)

$$\underbrace{x^2 + 6x + 9} + y^2 = 16$$

Hint: There is something special about <u>this</u> group

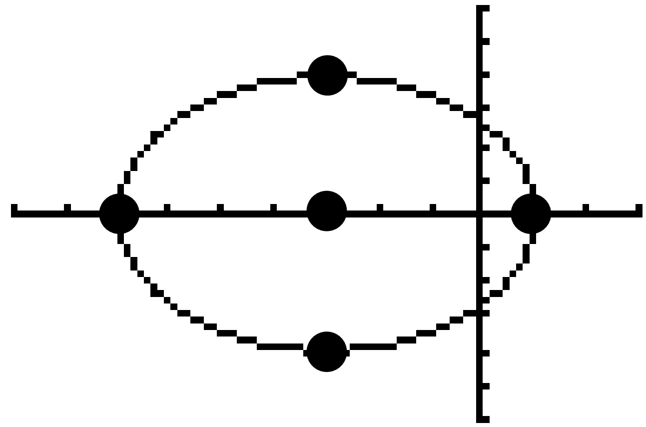
A simple factoring will give…

$$(x+3)^2 + y^2 = 16 = 4^2$$

Graph defaults to a FULL Circle

So **center** @ **(–3, 0)**

Radius = 4
so easy points to find on circle…
(1, 0) (–7, 0) (–3, 4) (–3, –4)



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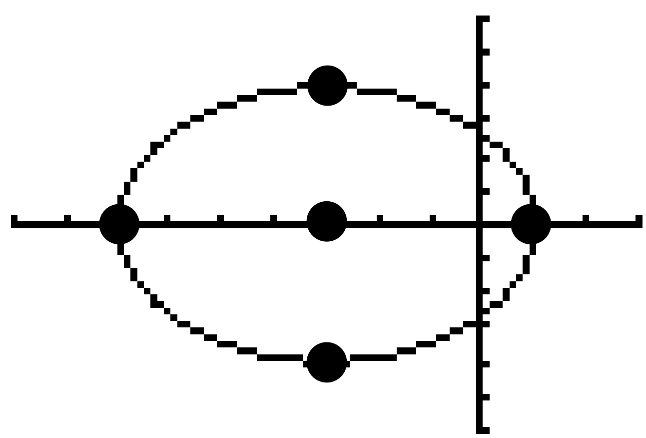

Circle Card #7

- → Remember that it is helpful to write the circle equation in standard form:

$$(x \pm a)^2 + (y \pm b)^2 = r^2$$

- → Identify and label the center.
- → Identify the radius and use it to label one point on the circle
- → Will the final graph be a FULL circle or PART of a circle? (it depends on what you start with)

$$x = -\sqrt{16 - y^2}$$

Circle Card #7

- → Remember that it is helpful to write the circle equation in standard form:

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- → Identify and label the center.
- → Identify the radius and use it to label one point on the circle
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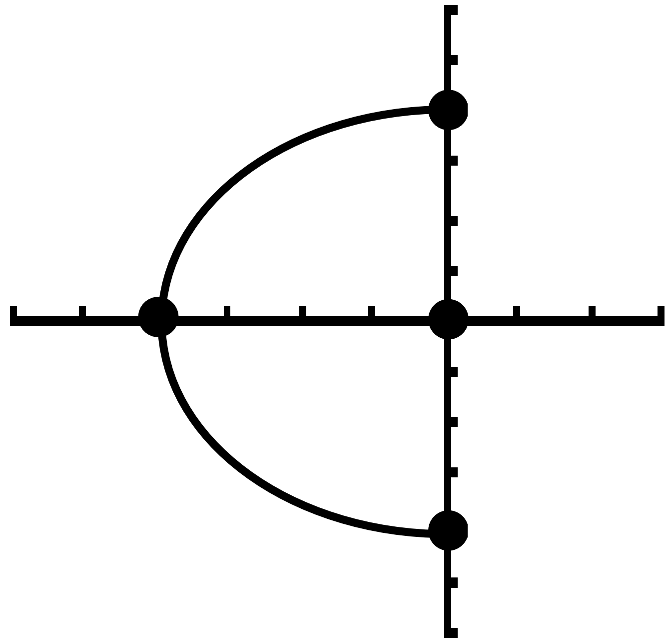
$$x = -\sqrt{16 - y^2}$$

Putting the equation in standard form…

$x^2 = 16 - y^2$

$x^2 + y^2 = 16 = 4^2$ the negative just gets "squared away"



We only get the negative ***left*** half of the circle since ***original form*** was given as x = $-\sqrt{}$

So **center @ (0, 0)**

Radius = 4
so easy points to find on circle…
(–4, 0) (0, –4) (0, 4)

Calculator Note: It would be difficult to get your calculator to graph this half circle since it is NOT a function… you actually have to type it in as piece wise functions… or just forget the calculator all together and do NO CALCULATOR math!!

Putting the equation in standard form…

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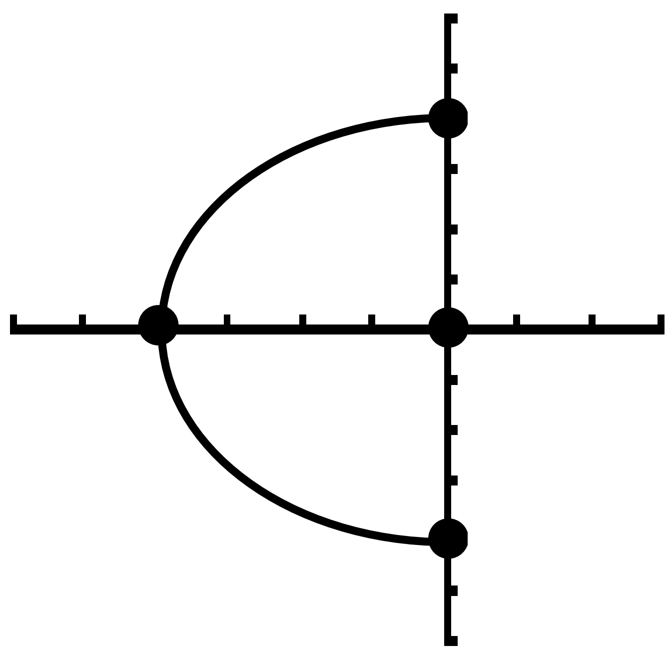

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REVIEW CARd #8

- → Create a graph of the following polynomial equation ***with out a calculator.***
- → Remember to consider using / thinking about…
  - ✓ roots
  - ✓ multiplicity and x axis behavior
  - ✓ degree and end behavior
  - ✓ the value of the y intercept

$$P(x) = -x(x-4)(x+5)^2$$

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  - ✓ roots
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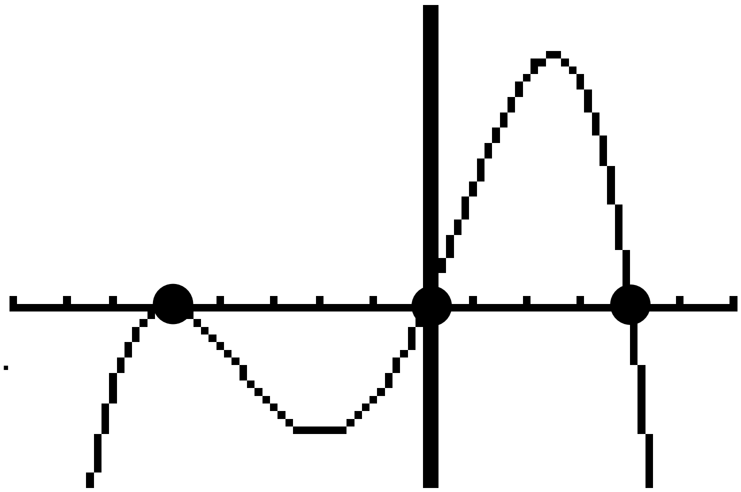
$$P(x) = -x(x-4)(x+5)^2$$

Equation contains x = 0, 4 & –5 as roots

x = –5 has an <u>even</u> multiplicity of two
so graph will just "touch" the x axis

Leading coefficient is negative, so
right side of graph will go down.

Equation has an overall <u>even</u> degree of 4.
so end behavior will be the SAME
on the right and left



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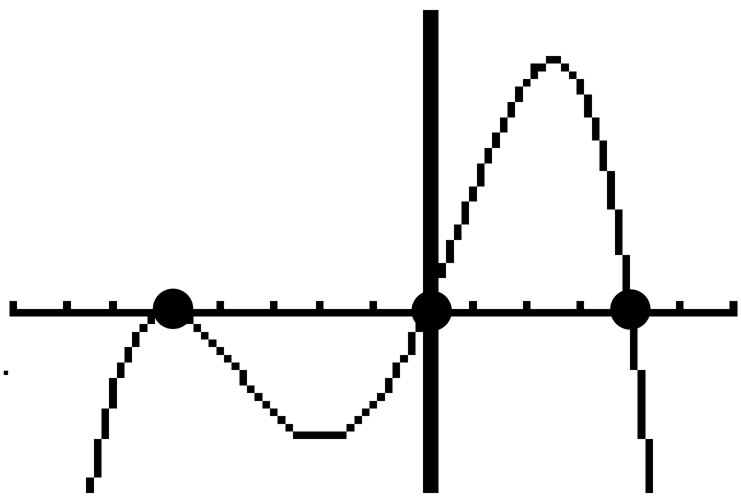

REVIEW CARD #9

→ It will be to your advantage to remember and be able to work with TRIG graphs during Calculus next year. Let's see what you remember.

→ Without using a calculator, identify the

- ✓ amplitude
- ✓ vertical shift
- ✓ y intercept
- ✓ max / min lines
- ✓ 2 or 3 x-intercepts if the period is 360 or 2π

$$f(x) = 2\sin x + 3$$

Hint: The parent sine wave starts at (0,0), then proceeds upward. Where then will this sine wave start? And how high/low will it go considering the amplitude and vertical shift.

REVIEW CARD #9

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- ✓ amplitude
- ✓ vertical shift
- ✓ y intercept
- ✓ max / min lines
- ✓ the values of 2 or 3 max/mins if the period is 360 or 2π

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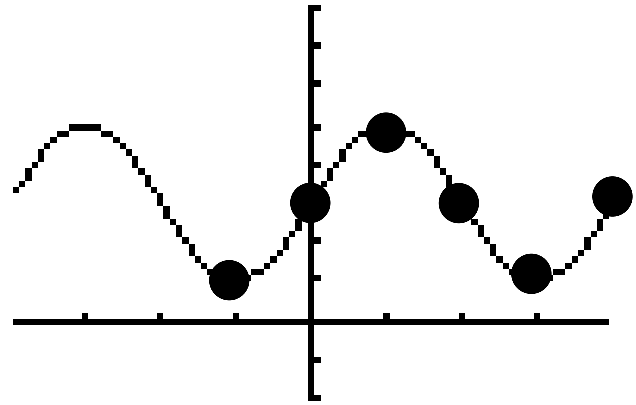
Hint: The parent sine wave starts at (0,0), then proceeds upward.
Where then will this sine wave start? And how high/low will it go considering the amplitude and vertical shift.

Since the vertical shift is +3, the original
y intercept of (0,0) will now be starting at (0,3)

The amplitude will stretch the graph 2 units above
and below this midline of y=3

So maxs will be at y = 5
and mins @ y = 1

A full period takes the graph to the point x = 360° y = 3
Half a period takes the graph to the point x = 180° y = 3
Therefore we have max / mins at
90° intervals @ (90°, 5) and (270°, 1)



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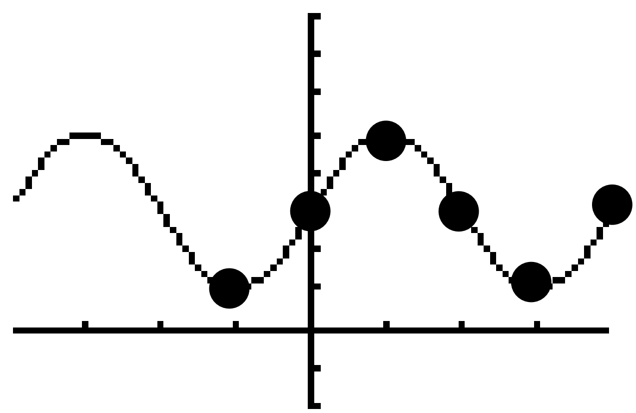

REVIEW CARD #10

→ It will be to your advantage to remember and be able to work with TRIG graphs during Calculus next year. Let's see what you remember.

→ Without using a calculator, identify the

- ✓ horizontal shift
- ✓ period
- ✓ 2 asymptotes
- ✓ 2 x-intercepts

$$f(x) = \tan(x - \tfrac{\pi}{2})$$

Hint: Don't be alarmed by the $\frac{\pi}{2}$ … this is simply a horizontal shift of 90° _____ Remember that the parent tangent graph starts @ (0,0) and the next asymptote is half a period away… of course the period of tangent is 180° or π

➔ So go ahead and try to draw some tangents, ***then*** label the features.

REVIEW CARD #10

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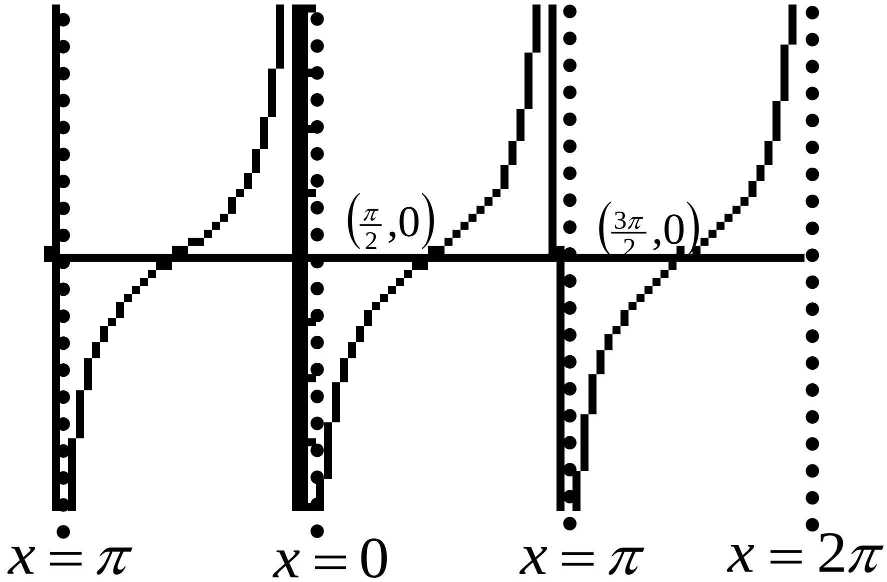
➔ So go ahead and try to draw some tangents, ***then*** label the features.

Horizontal Shift is $\frac{\pi}{2}$ units to the right,
so the first positive intercept will be located at $\left(\frac{\pi}{2},0\right)$

The next intercept will be a full period away @ $\left(\frac{3\pi}{2},0\right)$

The asymptotes are half way between the intercepts and are also a full period apart, so we could locate the first positive asymptote @ x= π and the next @ x = 2π

There is also an asymptote @ x = 0



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